Newton as Geodesist

The Problem of the Earth's Figure and the Argument for Universal Gravitation

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Isaac Newton's *Philosophiæ Naturalis Principia Mathematica* is usually remembered for its theoretical advances in mechanics and its empirical argument for universal gravitation. This argument, in turn, is taken to rest on Newton's empirical predictions of astronomical phenomena in the Principia's third book. For all its merits, this focus on astronomy has led many historians, physicists, and philosophers to overlook an equally important empirical problem in the *Principia*: deriving the earth's figure and the variation of gravity on its surface. In what follows, I hope to rectify this situation. After presenting a new reconstruction of Newton's often overlooked derivation of the earth's figure and surface gravity, I illustrate why these results were of primary importance to his argument for universal gravitation. This offers the first complete reconstruction of Newton's derivation *and* its methodological significance.[1]

The Derivation

Newton began his work on the derivation of planetary figures after learning about an experimental result obtained by French physicist Jean Richer. Richer had traveled to Cayenne in 1671 and conducted a series of experiments with a pendulum clock. Originally calibrated to Parisian astronomical time (48°40' latitude), Richer found the clock to lose an average of 2 ½ minutes per day relative to astronomical time in Cayenne (5° latitude) . This result was suprising for contemporaries but had an intuitive explanation in terms of the theory of

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[1] So far, the only published discussions of the work's methodological importance can be found in Schliesser and Smith 2000 and Smith 2014, who do not include reconstructions of Newton's derivation. Todhunter 1873 and Greenberg 1996 gives a discussion of the derivation in plain English, without any methodological contextualisation. Chandrasekhar 2003 also gives a reconstruction of the derivation in modern algebra, which contains some minor but quite confusing mistakes and ambiguities.

centrifugal motion recently developed by Christian Huygens. *Given that the centrifugal effect is strongest at the equator, one could expect a reduction in net effective surface gravity as one moves from Paris to Cayenne* (Huygens 1690, 146). While Newton accepted Huygens's theory, he realised that this explanation has an unintended empirical consequence. *If the earth is a sphere but its centrifugal effect is strongest at the equator, one would not only observe surface gravity variations, but the ocean would bulge up at the equator* – a proposition that Newton considered absurd. To avoid the unwarranted implication, he proposed that the solid earth had *behaved approximately like a malleable fluid* throughout its formation, gradually bulging up at the equator in response to the centrifugal effect. Hence, Newton proposed to model planets as rotating fluids in equilibrium, where the shape of a planet must be stable under the joint action of the forces resulting from the rotational motion and gravitational attraction between the constituent particles of a rotating fluid (C&W, 470).

Newton's first step in deriving the earth's empirical parameters from this theory was to calculate the ratio between the strength of gravitational acceleration and centrifugal force at the equator (1/290.8) based on the period of the earth's diurnal rotation and estimates of its equatorial diameter. Newton knew about the length of two meridional arcs that could be used for such a calculation, measured by Richard Norwood in England and Jean Picard in France. The second empirical input Newton needed was the magnitude of gravitational acceleration at the equator. He calculated this value by extrapolating from Richer's pendulum measurements at 48°50' latitude to the corresponding value at the equator of a homogenous sphere (C&W, 826)

Equipped with an empirical ratio between centrifugal effect and equatorial surface gravity, Newton faced a tough problem: How could one express mathematically what it means for a rotating fluid whose constituents attract according to a certain law of gravity to be in a state of equilibrium? Newton by employing up with an ingenious thought experiment, which he had originally developed in his 1685 *Liber Secundus* manuscript (Newton 1685, prop. 50): a rotating body is in a state of hydrostatic equilibrium if the weight of water in two channels X and Y, where X connects the equator to the earth's center and Y the earth's center to one of the poles, is identical. Since X is affected by the centrifugal effect, this condition is fulfilled if the overall centrifugal "pull" on equatorial particles is compensated by a change to the figure. In other words, the equatorial regions need to "bulge up" and the poles "flatten" to such an extent that the total weight of X and Y (i.e., net quantity of gravitational attraction towards the center) is the same. This stipulative attempt to define hydrostatic equilibrium later came to be known as the "principle of balanced columns" (Todhunter 1873, 1:14).

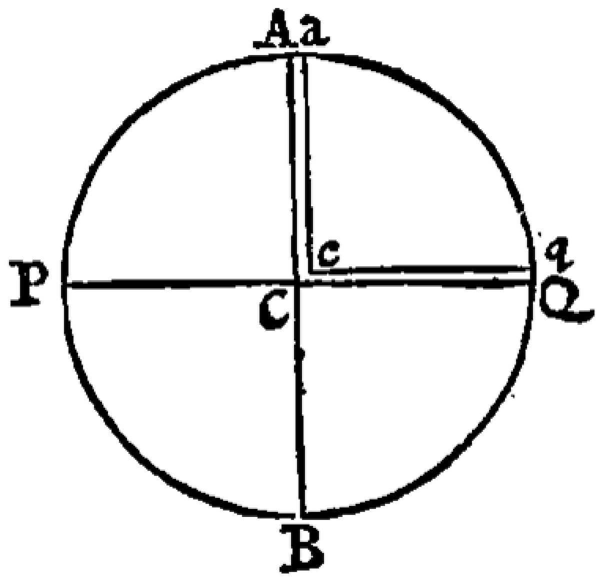


Fig. 1: Newton's illustration of hydrostatic equilibrium through the "principle of balanced columns" in the first edition of the Principia. Taken from: Isaac Newton, *Philosophiæ Naturalis Principia Mathematica*, London 1687, 422.

Newton then used his theory of gravitational attraction to derive the figure that a rotating body would need to acquire to balance the net attraction on the two columns. More precisely, he determined the ratio between equatorial diameter and polar axis that fulfils this equilibrium condition. Newton begins his derivation by determining the ratio between polar and equatorial surface gravity for the simple case of a non-rotating oblate figure, with the axis-diameter ratio of 100/101, arriving at 501/500.[2] If this result is multiplied by the length of the two fluid columns (100/101), we obtain the ratio of 501/505 between the net gravitational forces acting on the polar and equatorial fluid columns. Therefore, the net gravitational force acting on the equatorial fluid column is greater than the corresponding net force acting on the polar fluid column by a magnitude of 4/505. *It follows that if a spheroid with the dimensions of 100/101 would be rotating, the ratio between equatorial surface gravity and centrifugal force must be 4/505 for it to be in a state of hydrostatic equilibrium.* Since Newton's premise was that the earth de facto *is* in a state of hydrostatic equilibrium, he then extended this thought experiment from the simple case to the empirically parameters of the physical earth. For his previously determined 1/290.1 ratio between equatorial surface gravity and centrifugal force, he calculated a corresponding polar axis and equatorial diameter ratio of

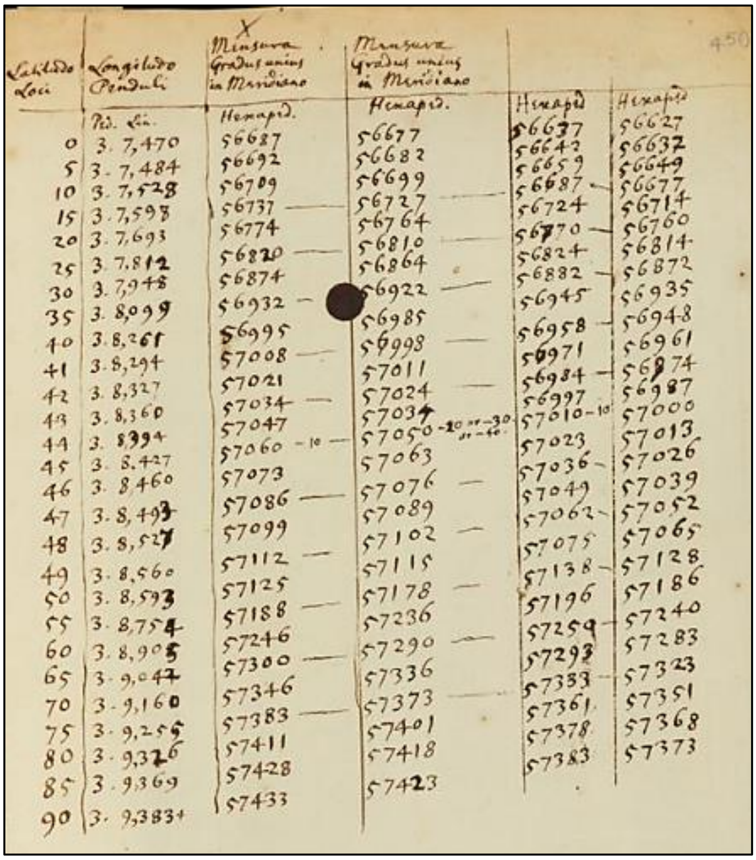
[2] Newton's derivation for the attraction of perfectly spherical and spheroidal compound bodies is given in Book 1, Prop. 91, coroll. 1 and 2 and reconstructed in its original notation in the appendix.

689 to 692. He concludes *that the earth, modeled as a homogenous spheroid that rotates with uniform angular velocity, must have polar and equatorial axes with a length ratio of 689 to 692 to be in a state of hydrostatic equilibrium.*

Corresponding to the resulting latitudinal variation in the distances between points on the earth's surface and its center, he then calculated the effective surface gravity – indicated by the length of the seconds-pendulum – to vary as the square of the sine of the latitude (C&W, 473). By deriving a *general* latitudinal variation, he is no longer just concerned with the length ratio between the equatorial diameter and polar axis but explicitly committed to a model of the earth's overall figure. Namely, that of an oblate ellipsoid with an ellipticity of 3/692 (~1/230.7). Using the measured pendulum length at 48°40' astronomical latitude in Paris as a reference point (3ft 17/24 inches), he predicted that the pendulum has to be shortened by 81/1000 and 89/1000 inches in Gorée (14°15') and Cayenne (5°) to preserve its period – two reasonably close but still noticeably inaccurate approximations of the existing measurements in those locations (100/1000 and 125/1000 inches).

The Earth's Figure and the Argument for Universal Gravitation

So far, I have demonstrated that Newton invested a considerable amount of effort in deriving the earth's figure and latitudinal variation in surface gravity. Besides a novel definition for the hydrostatic equilibrium of rotating bodies, these results presumed Newton's theory of gravitational attraction. His empirical predictions only hold if the mutual attraction between *all* constituent particles of the earth is assumed throughout the derivation. Hence, these predictions offered a test for the most fundamental and novel assumption in Newtons theory of gravitation. Namely, that gravity acts "universally" between all particles of matter. In fact, as George Smith has shown, the geodetic predictions offer the *only* such test in the *Principia* (Schliesser and Smith 2000, 5). It is clear from archival sources that Newton was sharply aware of this situation. Not only did his editor Roger Cotes keep pushing him to revise the geodetic results in light of new data (Cotes to Newton 1712, 234-235), but Newton explicitly revised his value for the earth's ellipticity from 689/692 to 1/230 and added a table with detailed predictions in the second edition of the Principia (C&W, 472). These predictions concerned the lengths of an isochronous pendulum with a period of one second at different latitudes (indicating surface gravity) and the length of 1° of meridian at different latitudes (indicating surface curvature). As is evident from his scribbles during the editing process, Newton again revised these predictions in the editorial process for the third edition (Fig. 2).



Latitudo Loci	Longitudo Penduli	Mensura Gradus unius in Meridiano	Mensura Gradus unius in Meridiano		
	Ped. Lin.	Hexaped.	Hexaped.	Hexaped.	Hexaped.
0	3. 7,470	56687	56677	56637	56627
5	3. 7,484	56692	56682	56642	56637
10	3. 7,528	56709	56699	56659	56649
15	3. 7,598	56737	56727	56687	56677
20	3. 7,693	56774	56764	56724	56714
25	3. 7,812	56820	56810	56770	56760
30	3. 7,948	56874	56864	56824	56814
35	3. 8,099	56932	56922	56882	56872
40	3. 8,261	56995	56985	56945	56935
41	3. 8,294	57008	56998	56958	56948
42	3. 8,327	57021	57011	56971	56961
43	3. 8,360	57034	57024	56984	56974
44	3. 8,394	57047	57037	56997	56987
45	3. 8,427	57060 -10	57050 -20 or -30 or -40	57010 -10	57000
46	3. 8,460	57073	57063	57023	57013
47	3. 8,493	57086	57076	57036	57026
48	3. 8,527	57099	57089	57049	57039
49	3. 8,560	57112	57102	57062	57052
50	3. 8,593	57125	57115	57075	57065
55	3. 8,754	57188	57178	57138	57128
60	3. 8,905	57246	57236	57196	57186
65	3. 9,047	57300	57290	57259	57240
70	3. 9,160	57346	57336	57293	57283
75	3. 9,255	57383	57373	57333	57323
80	3. 9,326	57411	57401	57361	57351
85	3. 9,369	57428	57418	57378	57368
90	3. 9,383	57433	57423	57383	57373

Fig. 2: Newton's repeatedly revised predictions of the pendulum (column 1) and arc lengths (column 2-4) at different latitudes; draft from between the second and third edition. Cambridge University Library, Newton Manuscripts, MS Add 3965, 450r.

The obvious question to ask at this point is whether Newton's geodetic predictions could live up to their enormous importance in the argument for universal gravitation. On a naïve reading, the answer is "no". When the third and last edition of the *Principia* was published, Newton had access to one measurement of the latitudinal variation in the length of 1° of meridian and five pendulum measurements of the variation of surface gravity with latitude. The arc measurement entirely disagreed with Newton's predictions, seemingly indicating that the earth is an oblong – rather than oblate – spheroid. Out of several existing pendulum measurements, only Jean Richer's mentioned above seemed broadly admissible, still markedly disagreeing with Newton's prediction. The situation is similarly dire when we turn to current data, as satellite measurements give the earth's ellipticity as 1/298.257223563 (WGS 84).

Taking such a pessimistic view on the fruits of Newton's geodetic work misses some of the most important nuances of the Principia. As George Smith has forcefully argued, Newton's Principia does not only propose theoretical predictions, but a distinctive methodology of *testing through approximation.* Newton took for granted that his initial predictions are likely to be inaccurate because he relied on several uncertain background hypotheses when deriving them. The success of the universal theory of gravitation, then, was never intended to be measured by the immediate agreement between initial predictions and measurements. Rather, Newton intended his theory to be tested based on how well it could guide *consecutive adjustments of background hypotheses that lead to a convergence among measurements* (Smith 2014).

Hence, Newton did not aim to establish the ellipticity and latitudinal surface gravity variation of the earth once and for all. Rather, they constituted first approximations, which should allow for empirical adjustments of the uncertain assumptions in the derivation of the earth's equilibrium figure. The most obvious idealization in that derivation was the assumption that the rotating earth has a *homogenous* density. Indeed, it is precisely this assumption that Newton modified in the first edition of the *Principia*, reacting to the disagreement between his predictions and Richer's measurements in Cayenne and Gorée. If the earth is denser at its center, he insinuated, the ellipticity of the earth's equilibrium figure and, ipso facto, its surface gravity variation will differ:

> These differences are a little greater than the differences $\frac{81}{1000}$ and $\frac{89}{1000}$ which resulted from the above computation, and therefore (if one can have enough confidence in these rough observations) the earth will be somewhat *higher at the equator than according to the above calculation and denser at the center than in mines near the surface* (C&W, 473, my italics)[3]

This conclusion passes the torch to future researchers, inviting them to hypothesize different density distributions that accord with these initial measurement outcomes, which can then be further investigated based on increasingly precise empirical measurements (C&W, 473).

In line with this diachronic methodology outlined by Newton, geodesists eventually produced convergent measurements of the earth's ellipticity based on latitudinal surface gravity and curvature variations. For about two and a half centuries, they used the theories of gravitation and hydrostatic equilibrium to model the dependencies between the earth's figure, motion, and constitution, and gradually revised these parameters in the light of new measurements. By 1909, all major ellipticity measurements converged within 297.6±0.9, implying an inwardly increasing density (Ohnesorge forthcoming). By 1926, Viennese astronomer Samuel Oppenheim could conclude that these results offered overwhelming evidence for Newtonian

[3] Newton turned out wrong about the exact relationship between the earth's surface gravity, equilibirium figure, and the earth's interior density distribution. An increase in density towards the centre would imply that the earth's ellipticity and surface gravity variation *decrease*. This was first pointed out by Alexis Clairaut (1738), who subsequently proved it in his 1743 magnum opus *Theorie de la figure de la terre*.

gravity at a terrestrial scale, vindicating both Newton's theory of gravitation and his diachronic methodology (Sommerfeld and Oppenheim 1926, 122–23).

Appendix: Newton's derivation of the ratio between the gravitational accelerations at the equator and the pole of an oblate spheroid[4]

Newton argued that gravitational attraction varies as the inverse square of the distance and proportionally to the product of the masses between point particles with a certain mass:

$$F_g \propto \frac{M_1 M_2}{r^2}$$

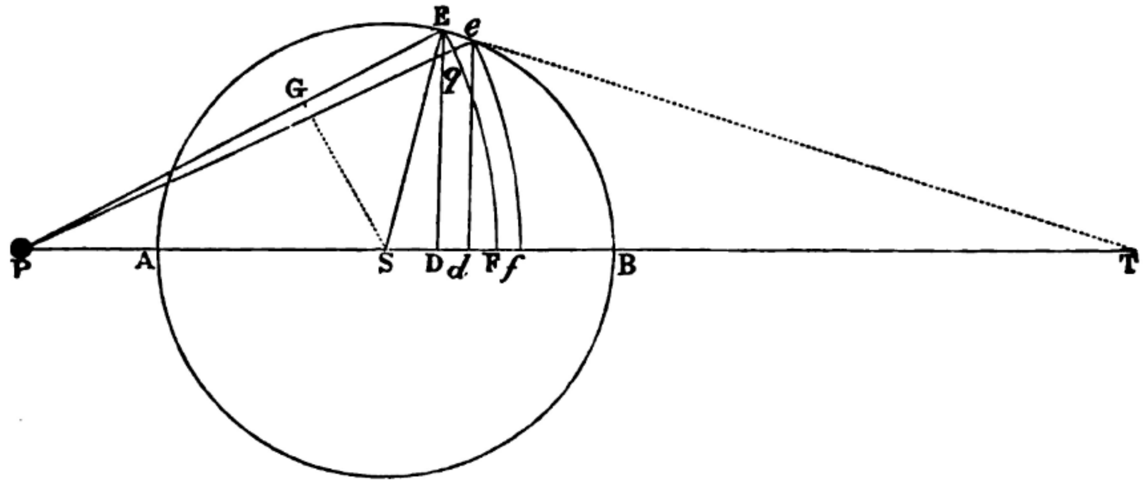
Where F is the gravitational force, M_1 and M_2 are the masses of two attracting point particles, and r is the distance between them.

When deriving the earth's figure, Newton needed to relate this exceedingly abstract equation to the ratio between the gravitational acceleration at the equator and poles of the physical earth. As we saw in section 3, this problem first needed to be solved for the simpler case of a non-rotating homogeneous oblate spheroid, from which Newton then derived the figure and surface gravity of a rotating homogeneous oblate spheroid (representing the earth). One quickly realises the main difficulty in this endeavour: a spheroid is not a point particle but composed of several particles, requiring a method to calculate their net attraction. In a first approximation, the spheroid may be treated as a homogenous sphere HS around a centre S. Newton had already proved earlier in book 1 that (i) the attraction outside of a homogenous sphere varies as if its mass is concentrated at one point at its centre and (ii) attraction inside a homogenous sphere varies linearly with distance from the centre.

Treating the earth as a sphere, the problem then becomes **how derive the quantity of net attractive force that HS exerts on an external point particle P**. Assuming that mass difference between HS and P is sufficiently great, the second law of motion implies that this quantity is directly proportional to the gravitational acceleration on the surface of HS. Newton approaches this problem by calculating the attraction of a point particle E on the surface of HS and a two-dimensional zone Eq inside of the sphere (prop. 79), before he

[4] This reconstruction incorporates elements from George Smith's summary of Newton's derivation of the attraction of spheres, as given in the lecture series "Newton's Principia" at Tufts University in the academic year 2020/21. The only existing reconstruction of the other parts of this derivation (attraction of spheroid, application to the earth's figure) is the algebraic version given by Chandrasekhar (2003). Note that Chandrasekhar's treatment is not only in a different notation but contains several mistakes. At one point, he argues that Newton had derived an ellipticity of 1/289 for the earth, while he had in fact derived an ellipticity of 1/230 (as noted in: Smith 1996). More consequentially, however, he repeatedly claims that Newton had used a model spheroid with an ellipticity of 1/100 in his derivation, while he is in fact using a model with an ellipticity of 1/101. This is particularly confusing since this value would have led Newton to a different conclusion. In fact, Newton does not even use the quantity of ellipticity in props. 19 and 20 but only refers to the length ratio between equatorial diameter and polar axis. In what follows, I stick closer to the original derivation and denote the dimensions of the different spheroids in terms of the ratio between their equatorial diameter and polar axis.

generalises the expression to account for the attraction for different three-dimensional shells, whose compounded attraction gives the net attraction of HS (prop. 80).



Newton begins by calculating the relationship between the gravitational attraction of a point particle E and the zone Eq. For this, he assumes that the distance between the arcs EF and *ef* vanishes and exploits the resulting co-variation in the length of different triangle sides in the above figure. The net attraction of Eq on P varies with the attraction of a point particle at E and several sides in the above figure as follows:

$$F_{P\to \mathrm{E}q} \propto F_{P\to E}(PE \times Dd)\frac{PD}{PE}$$

which can be further simplified as:

$$F_{P\to \mathrm{E}q} \propto F_{P\to E}(PD \times Dd)$$

From this Newton could calculate the attractive force of the surface of the shell FE as follows:

$$F_{P\to Shell\ surface} \propto F_{P\to E}(\frac{PF^2}{2} - \frac{PD^2}{2}) \propto F_{P\to E} DE^2$$

Generalising further, he arrived at an expression for the attraction of the volume of that shell

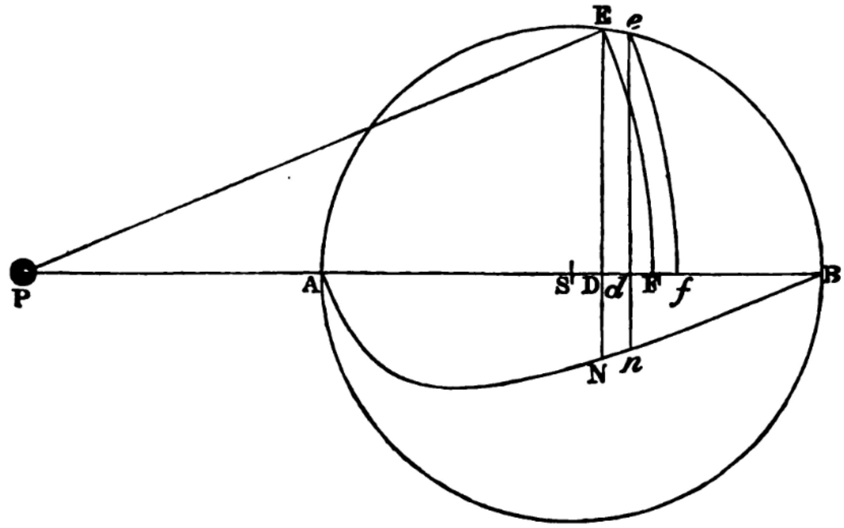
$$F_{P\to Shell\ volume} \propto F_{P\to E} \times DE^2 \times Ff$$

Where Ff can be defined in terms of already known ratios as follows:

$$Ff = \frac{PS}{PD} Dd$$

Now we know how to determine the attractive force that a spherical shell exerts on P based only the attractive force that a point particle at E exerts on P. The next task is to calculate the attraction of HS, which is taken to be constituted by a large number of such concentric shells with vanishings widths. Newton begins by representing the magnitude of attraction of one such shell by the line DN, where DN is drawn perpendicular to AB and its lengths varies as follows:

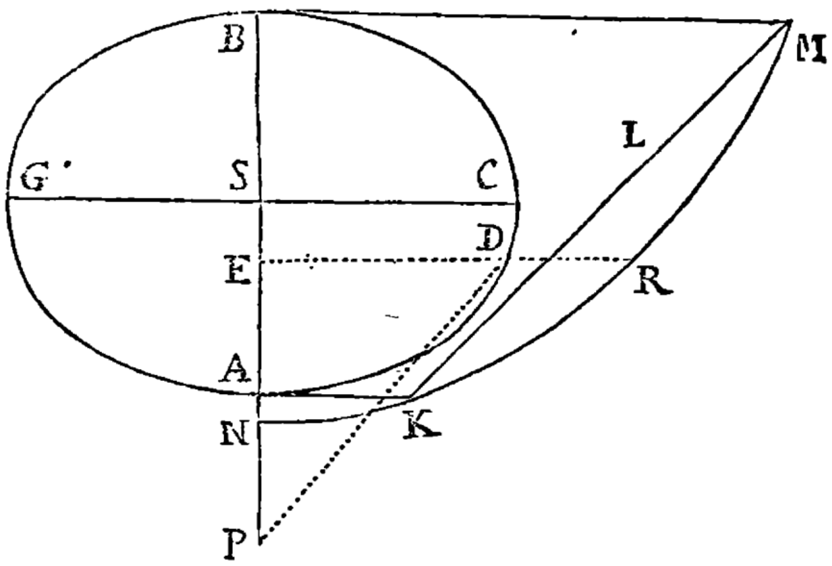
$$DN \propto F_{P\to E} DE^2 \frac{DE^2 \times PS}{PE}$$



The different shells can then be determined by integrating the curve ABN passing through the endpoints of multiple of such lines, which are each drawn analogously to DN:

$$F_{P\to HS} \propto \int_A^B DN \times DX = area\ (ABN)$$

So far, the earth has been treated as a sphere. The next step consisted in extending the previous results to the case of a homogenous oblate spheroid. Newton begins by (i) giving the attraction on an external particle P that lies on the polar axis AB of a homogenous oblate spheroid HOS (Book 1, Prop. 91, Corollary 2) and subsequently (ii) determines the ratio between the polar and equatorial attraction of a non-rotating oblate spheroid (Book 3, Prop. 19). Unfortunately, Newton's written treatment of (i) leaves "a long way from the conundrum-like fashion in which the attraction is expressed in the corollary to its value at the pole, on the axis" (Chandrasekhar 2003, 387). In his geometrical approach to (i), Newton draws a conic section NKRM that is geometrically connected to HOS as follows:



He then states that the attraction of HOS on P varies with the attraction of a sphere SAB that has a diameter of AB as follows:

$$F_{P\to HOS} \propto F_{P\to SAB}\left[\left(\frac{AS\times CS^2 - PS\times KMRK}{PS^2+CS^2-AS^2}\right):\left(\frac{AS^3}{3PS^2}\right)\right]$$

The above expression is the final result of the brief (~ one page) corollary that Newton cites in proposition 19 on the earth's figure in book 3. He argues that it implies the following statement about the ratio between attraction at the pole of a homogenous oblate spheroid HOS with an equatorial-diameter-to-axis ratio of 100/101 and the attraction at the pole of a homogenous sphere with the spheroid's equatorial axis as its diameter:

$$F_{Pole\to HOS\frac{100}{101}} = F_{Pole\ HS\frac{100}{100}} \times 126/125$$

The gap between the convoluted and unproven expression for $F_{P\to HOS}$ and this solution for $F_{Pole\to HOS\frac{100}{101}}$ motivated Chandrasekhar to argue that Newton had already developed a generalised algebraic theory for the attraction of any oblate spheroid, which he deliberately "withheld".[5] The gap might have also contributed its part to why many seventeenth-century

[5] These results are generally ascribed to MacLaurin. It might well be that Newton simply "got lucky" with a conjecture and did not yet have a proven a theorem analogous to MacLaurin's. Derek Whiteside notes that Newton published on the analysis of conic integral that *would* be applicable for this problem in 1704, which is after the first edition of the Principia but before its second and third edition (Newton 1974, 226–27). To the best of my knowledge, there is no decisive historical evidence to decide the issue.

physicists found this part of the Principia so incomprehensible, including notable figures like Daniel Bernoulli and Pierre Maupertuis (Terrall 2002, 56).

Be it via conjecture or "withheld" algebraic solution, Newton did arrive at the above expression for the surface gravity at the pole of HOS. To solve the initial problem of determining the ratio between gravitational acceleration at the equator and poles of HOS, however, we also need to know about the surface gravity at the equator of HOS. Newton does not provide such an expression in book 1. Rather, he derived the ratio between the gravitational acceleration at the poles and the equator through a simple and powerful argument, which we find in proposition 19 of book 3. His reasoning exploits the fact the quantity of volume increase of constructing a homogenous oblate spheroid HOS with an ellipticity *e* from a homogenous sphere *HS* is equal to the quantity of volume decrease of constructing a homogenous prolate spheroid HPS with the inverted ellipticity. Hence:

$$F_{Equator \to HOS\frac{100}{101}} = \frac{1}{2}(F_{Pole\ HPS\ 101/100} \times F_{HS\ \frac{101}{101}})$$

Newton notes the following ratios between the gravitational attraction at the poles and equator of the different figures that can be obtained in this manner:

$$F_{HS\frac{101}{101}} : F_{Equator \to HOS\frac{100}{101}} = 126/125.5$$

$$F_{HS\frac{100}{100}} : F_{HS\frac{101}{101}} = 100/101$$

To solve the initial problem of determining the ratio between $F_{Pole \to HOS\frac{100}{101}}$ and $F_{Equator \to HOS\frac{100}{101}}$, we now only need to multiply these two ratios with the previous results that Newton had conjectured (or derived via withheld analytic expression) for $F_{Pole \to HOS\ \frac{100}{101}}$. Hence, he calculates:

$$F_{Pole \to HOS\frac{100}{101}} : F_{Equator \to HOS\frac{100}{101}} = \frac{126 \times 126 \times 100}{12S \times 125.5 \times 101} = \frac{501}{500}$$

Therefore, $\frac{501}{500}$ is the ratio between the gravitational accelerations at the equator and the pole of a homogenous oblate spheroid with an axis-diameter ratio of $\frac{100}{101}$.

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